Section 6.1

Given: $f(x)=\frac{x^2+5x}{2x+5}$ The domain is $D_f \quad x \neq -\frac{5}{2}$ because we must not allow the denominator to be zero. A zero in the numerator is acceptable and makes the value of the fraction zero. A zero in the denominator is never acceptable.

Find $A)$ $f(0)$, B) $f(1)$, and C) $f(-2)$

$A)$	$B)$	$C)$
$f(0)=\frac{0+0}{0+5}$	$f(1)=\frac{1+5}{2+5}$	$f(-2)=\frac{4-10}{-4+5}$
$f(0)=0$	$f(1)=\frac{6}{7}$	$f(-2)=-6$

On part C, notice that we do NOT write the denominator one.

The fraction on the right side of the equal sign in our function above is called a rational expression.

Section 6.1 gives us the tools to simplify rational expressions.

The rules that are used for rational expressions are the same as those used for fractions found in arithmetic.

$$\frac{2}{3}\cdot\frac{5}{7}=\frac{10}{21}$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd} \qquad \frac{x+1}{x-2}\cdot\frac{x}{x+3}=\frac{x^2+x}{x^2+x-6}$$ as you would expect.

When you multiply fractions in arithmetic, you **reduce** before you multiply like this: $\frac{4}{15}\cdot\frac{25}{24}=$ $\frac{1}{3}\cdot\frac{5}{6}=\frac{5}{18}$ You would not get: $\frac{4}{15}\cdot\frac{25}{24}=\frac{100}{360}$ because you would still need to reduce and to reduce larger numbers is more difficult than reducing smaller numbers.

$$\frac{(x+1)(x+2)}{x^2}\cdot\frac{x^3}{(x+1)(x-2)}$$ reduces to: $$\frac{x(x+2)}{x-2}$$

Notice the factor, x in the numerator does NOT reduce with the x on the bottom. The x on the bottom is NOT a factor. It is a term in the expression $x-2$. You can ONLY reduce exactly the same factors that occur on the top and on the bottom!

Example:
$$\frac{x^2-4}{2x^2-3x-2}$$
$$\frac{(x-2)(x+2)}{(2x+1)(x-2)}$$
$$\frac{x+2}{2x+1}$$

This is called "Reducing the fraction". We *NEVER* "cancel". This is a shortcoming in the textbook. We *NEVER* use the method of canceling!!

We can REDUCE only identical factors.

$\frac{x}{x+3}$ The x on the bottom is not a factor so it cannot reduce with the x on the top.

$\frac{4x+3}{2}$ The 4 in the $4x$ is a term in the expression $4x + 3$ and NOT a factor of $4x + 3$ so we cannot reduce with the 2 on the bottom.

$\frac{4(x+3)}{2}$ We CAN reduce the 4 on top with the 2 on the bottom because both are factors.

We get: $2(x+3)$. Notice that we do NOT have the one in the denominator showing on the bottom.

Section 6.2

Adding fractions (or rational expressions)

Why is the lowest common denominator 12: $\frac{3}{4}+\frac{5}{6}$?

Look at the prime factors of the denominators: 2, 2 and 2, 3

"The LCD must have two 2s in it because this one has two 2s. The LCD must have a 2 and a 3 because this one has a 2 and a 3. We already have at least one 2 so all we need to add is a 3." Our LCD is 2, 2, 3. The product is 12 and that is the obvious LCD.

To continue the addition problem……

We draw ONE line and put the LCD below Thus: $\frac{\quad + \quad}{2^2 \cdot 3}$

Also note that I have put the plus sign from the original problem on the top.

Now we look at ***each term*** in the original addition problem and answer the question, "What does the new denominator have that the original one does not have?" We will multiply the original numerator by the answer to that question.

In this case, the first answer is a 3 giving us : $\frac{3 \cdot 3 + \quad}{2^2 \cdot 3}$

And the second time we ask the question the answer is 2 so we now have:

$$\frac{3 \cdot 3 + 5 \cdot 2}{2^2 \cdot 3}$$

Finally we have $\frac{9+10}{12}$ that becomes $\frac{19}{12}$.

LCD for algebraic expressions are put together in exactly the same way.

$\frac{2}{21x}+\frac{5}{3x^2}$ Our LCD is $3 \cdot 7x^2 = 21x^2$

For each item being added we ask: "What does the LCD have that this denominator does not have?" What ever is missing, we multiply times the existing numerator of the term.

The answer to the first question is an x so the numerator is 2x Thus:

$$\frac{2x \quad + \quad}{3 \cdot 7 \cdot x^2}$$

Notice the plus. That is the same sign that is used between the terms.

The answer to the question the second time is 7 so we multiply the 5 times the 7 and get $\dfrac{2x \quad + \ 35}{3 \cdot 7 \cdot x^2}$ and the final answer is $\dfrac{2x+35}{21x^2}$.

Example 9, page 367 *should* have been done as follows:

$$\frac{5x}{x-2y}-\frac{3y-7}{2y-x}$$
$$\frac{5x}{x-2y}-\frac{3y-7}{-(x-2y)}$$
$$\frac{5x}{x-2y}+\frac{3y-7}{x-2y}$$
$$\frac{5x+3y-7}{x-2y}$$

Example 7, page 367 *should* look like this:

$$\frac{2y+1}{y^2-7y+6}-\frac{y+3}{y^2-5y-6}$$

Factor the original. The first line now looks like this: $\dfrac{2y+1}{y^2-7y+6}-\dfrac{y+3}{y^2-5y-6}$

$$(y-6)(y-1) \quad (y-6)(y+1)$$

The LCD is $(y-6)(y-1)(y+1)$.

For the *second line of our problem*, we draw ***ONE line*** and put the LCD on the bottom: $\dfrac{\quad - \quad}{(y-6)(y-1)(y+1)}$. **Notice that the negative sign from the original problem is in place as well.**

"What does this LCD have that this first denominator does not have? ($y + 1$). Multiply times the existing numerator and move to the next fraction.

So far we have: $\dfrac{(2y+1)(y+1)-}{(y-6)(y-1)(y+1)}$

Again, ask the question and multiply the numerator by your response to get:

$$\frac{(2y+1)(y+1)-(y+3)(y-1)}{(y-6)(y-1)(y+1)}$$

Clear parenthesis --- partially ---

$\frac{y^2+3y+1-\left(y^2+2y-3\right)}{(y-6)(y-1)(y+1)}$ clearing the last of the parenthesis: $\frac{y^2+3y+1-y^2-2y+3}{(y-6)(y-1)(y+1)}$

Combine like terms: $\frac{y^2+y+4}{(y-6)(y-1)(y+1)}$

Example 10 page 368

$$\frac{2x}{x^2-4}+\frac{5}{2-x}-\frac{1}{2+x}$$

They have a couple of annoying items in this expression that we need to fix before we actually start with this:

$$\frac{2x}{x^2-4}+\frac{5}{-(x-2)}-\frac{1}{x+2}$$

$$\frac{2x}{x^2-4}-\frac{5}{x-2}-\frac{1}{x+2}$$

Now we factor and decide on our LCD: $\frac{2x}{(x-2)(x+2)}-\frac{5}{x-2}-\frac{1}{x+2}$

We draw ONE line and put our LCD below: $\frac{\quad - \quad -}{(x-2)(x+2)}$.

Notice that we have our two negative signs in place.

As before, we ask the question. For the first term nothing else is needed, for the second we need an (x + 2) and the third needs an (x – 2) Thus:

$$\frac{2x \quad - \quad 5(x+2) \quad - \quad (x-2)}{(x-2)(x+2)}$$

Because of the negative signs, we **partially** clear parentheses then combine like terms and factor the result once again:

$$\frac{2x \quad - \quad (5x+10) \quad - \quad (x-2)}{(x-2)(x+2)}$$

$$\frac{2x \quad - \quad 5x-10 - \quad x+2}{(x-2)(x+2)}$$

$$\frac{-4x-8}{(x-2)(x+2)}$$

$$\frac{4(x+2)}{(x-2)(x+2)}$$

$$\frac{4}{x-2}$$

Section 6.3

Several of the examples presented in this section appear to be much more complicated that is necessary. The following is what the examples should have looked like.

Example 1, page 373 should look like this:

$$\frac{\frac{1}{a^3b}+\frac{1}{b}}{\frac{1}{a^2b^2}-\frac{1}{b^2}}$$

To work on a ***complex rational expression***, we work ONLY with the numerator ***until it is a fully factored, single fraction***. Once that is achieved, we go back and do the same for the denominator. Finally we MULTIPLY the numerator times the reciprocal of the denominator. Remember that we REDUCE before we multiply, if possible.

The numerator becomes: $\frac{1+a^3}{a^3b}$ and the denominator becomes: $\frac{1-a^2}{a^2b^2}$.

We multiply: $\frac{1+a^3}{a^3b}\cdot\frac{a^2b^2}{1-a^2}$ first we factor: $\frac{\overset{(1+a)\left(1-a+a^2\right)}{1+a^3}}{a^3b}\cdot\frac{a^2b^2}{\underset{(1-a)(1+a)}{1-a^2}}$

Reducing as we can: $\frac{\overset{(1+a)\left(1-a+a^2\right)}{1+a^3}}{\underset{a}{a^3b}}\cdot\frac{\overset{b}{a^2b^2}}{\underset{(1-a)(1+a)}{1-a^2}}$

$$\frac{b\left(1-a+a^2\right)}{a(1-a))}$$

Compare to page 374.

Example 3, $\frac{\frac{3}{x}-\frac{2}{x^2}}{\frac{3}{x-2}+\frac{1}{x^2}}$.

As before we assemble the numerator and denominator: $\frac{3x-2}{x^2}$ and $\frac{3x^2+x-2}{x^2(x-2)}$.

Multiply: $\frac{3x-2}{x^2}\cdot\frac{x^2(x-2)}{3x^2+x-2}$

$$\frac{3x-2}{x^2}\cdot\frac{x^2(x-2)}{3x^2+x-2}$$

Factor and reduce:…

$$\frac{3x-2}{x^2}\cdot\frac{x^2(x-2)}{(3x-2)(x+1)}$$

$$\frac{(x-2)}{(x+1)}$$

Example 5 page 378

$$\frac{a^{-1}+b^{-1}}{a^{-3}+b^{-3}}$$

$$\frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a^3}+\frac{1}{b^3}}$$

$$\frac{\frac{b+a}{ab}}{\frac{b^3+a^3}{a^3b^3}}$$

$$\frac{b+a}{ab}\cdot\frac{a^3b^3}{b^3+a^3}$$

$$\frac{b+a}{ab}\cdot\frac{a^3b^3}{(b+a)(b^2-ab+a^2)}$$

$$\frac{b+a}{ab}\cdot\frac{a^3b^3}{(b+a)(b^2-ab+a^2)}$$

$$\frac{a^2b^2}{(b^2-ab+a^2)}$$

Section 6.4

Solving equations involving rational expressions is much like simplifying rational expressions as we did in section 6.3.

The key is the LCD. The quote is : "What does this denominator have that this denominator does not have?"

Example 1, page 382

$\frac{x+4}{3x}+\frac{x+8}{5x}=2$ This is line 1 of the problem

The LCD is 15*x*. $15x\left(\frac{x+4}{3x}+\frac{x+8}{5x}=2\right)$ This is the also the original line with parentheses and the LCD written.

Line 2: $5(x+4)+3(x+8)=30x$. Clear parentheses.
Line 3: $5x+20+3x+24=30x$ Combine like terms and solve.

Example 2 pg 383

$$\frac{x-1}{x-5}=\frac{4}{x-5}.$$

$$(x-5)\left(\frac{x-1}{x-5}=\frac{4}{x-5}\right)$$
$$x-1=4$$
$$x=5$$

BUT we notice that the 5 would cause a zero in the denominator therefore, we reject the 5. There is no solution to this equation.

The other examples are worthwhile and should be read.

The methods shown in our textbook is not the best method for simplifying or solving the problems in sections 6.3 and 6.4.

Replace the method shown in the text with the following:

Example 1: Simplify: $\frac{x}{x+1}+\frac{2}{3}$ Example 2: Solve: $\frac{x}{x+1}=\frac{2}{3}$

Before actually showing the work, we need to point out the difference between the two examples.

Example 1 is and expression. Example 2 is an equation. The work used on both is similar but there are some very important differences.

First I will show Example 1:

Line 1 $\frac{x}{x+1}+\frac{2}{3}$ original problem Notice NO = Sign!!

Line 2 (start) $\frac{\quad + \quad}{3(x+1)}$ Only ONE fraction bar is allowed. Notice LCD on the bottom.

We use the "we need a $x+1$ because this one has an $x+1$; we need a 3 because this one has a 3."

Line 2 (adjust numerator) $\frac{x(3)+2(x+1)}{3(x+1)}$ "What does the denominator have that the first fraction does not have? A 3. So we put a 3 times the x that is on the top of the first fraction. What does the denominator have that the second fraction does not have? A $x+1$ so we multiply the 2 times the $+ 1$."

Line 3 $\frac{3x+2x+2}{3(x+1)}$ We cleared the parentheses on the top.

Line 4 $\frac{5x+2}{3(x+1)}$ We would factor the numerator if possible and reduce if possible.

When you do problems that are similar to Example 1, you are allowed ONLY ONE fraction bar on line 2 of your problem. Failure to follow this instruction forfeits most, if not all, of the points possible on the problem.

Example 2 on the next page.

Example 2 $\frac{x}{x+1}=\frac{2}{3}$ Original problem HAS = sign. This is an equation

Line 1 (original problem) $\frac{x}{x+1}=\frac{2}{3}$

Line 1 (start) $\left(\frac{x}{x+1}=\frac{2}{3}\right)$ Put ONE set of parentheses around the entire equation

Line 1 (LCD) $\left(\frac{x}{x+1}=\frac{2}{3}\right)\left(3(x+1)\right)$ The $3(x+1)$ is the LCD. The LCD is determined in exactly the same manner as described in example 1 above.

We will multiply the LCD times each term in our equation.

When we multiply the $3(x+1)$ times $\frac{x}{x+1}$, the x +1 reduces leaving only the 3 and the x.

When we multiply the $3(x+1)$ times the $\frac{2}{3}$, the threes reduce leaving the 2 and the $(x+1)$.

Line 2 $3x=2(x+1)$ Notice: NO FRACTIONS!!!

Line 3 $3x=2x+2$ Clear parentheses

Line 4 $x=2$ Solve the equation

When you are solving equations that contain fractions and those fractions have variables in the denominators, you MUST have cleared all fractions by the second line of the problem.